Exercise 42

Each limit represents the derivative of some function f at some number a. State such an f and a in each case.

$$\lim_{\theta \to \pi/6} \frac{\sin \theta - \frac{1}{2}}{\theta - \pi/6}$$

Solution

Recall that the derivative of $f(\theta)$ is defined by

$$f'(\theta) = \lim_{\theta \to a} \frac{f(\theta) - f(a)}{\theta - a}.$$

Comparing this to the given limit,

$$f(\theta) = \sin \theta,$$

and its derivative is being evaluated at $a = \pi/6$.